MATH 2418

Exam 2 Reviews for the grade of A-

4.4

**Book**

Chapter main points

1. Columns of a matrix are orthonormal
   1. The dot product of each column with itself results in identity matrix
   2. Remember dot product of any matrix is the column’s transpose times column
   3. QT(Q) = I

|  |  |  |
| --- | --- | --- |
| -- | q1T | -- |
| -- | q2T | -- |
| -- | q2T | -- |

|  |  |  |
| --- | --- | --- |
| | | | | | |
| Q1 | Q2 | Q3 |
| | | | | | |

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

QT(Q) = =

1. Orthogonal Matrix requires:
   1. Q is square
   2. Q(QT) = I
   3. QT = Q inverse
2. Least squares solution to Qx = b:
   1. Remember x hat = QT(b)
   2. Projection of b:
   3. p = QQTb = Pb
3. Gram-Schmidt
   1. Find linearly independent vectors first
   2. Follow formula to find orthogonal A, B and C
      1. A = a
         1. A can be anything, we calculate B and C from this value
      2. B = b – [AT(b)/AT(A)] (A)
      3. C = c – [AT(c)/AT(A)] (A) – [BT(c)/BT(B)] (B)
   3. Normalize A, B and C
      1. Q1 = A / ||A||
      2. Q2 = B / ||B||
      3. Q3 = C / ||C||
4. Any q(i) calculated in Gram-Schmidt can be calculated from the projection the corresponding a(i).
   1. a(i) = [a(i) – p(i)] / || ai – pi||
   2. p(i) = [a(i)Tq(1) ]q(1)+ … + [a(i)T q(i-1) ]q(i-1)
5. Each a(i) will be a combination of q(1) to q(i).
   1. A = QR
   2. Q is orthogonal
   3. R is triangular

Facts

Orthonormal basis vectors - the columns of Q after Gram-Schmidt

Orthogonal – dot product 0

Orthonormal – normalized orthogonal vector (length 1)

Rules

(Qx)T = xTQT = swap order and transpose both

**Class**

**Recitation**